Exercise 14

Use the definition of continuity and the properties of limits to show that the function is continuous at the given number a.

$$f(x) = 3x^4 - 5x + \sqrt[3]{x^2 + 4}, \quad a = 2$$

Solution

By definition, a function is continuous at a number a if

$$\lim_{x \to a} f(x) = f(a).$$

Evaluate the function at $x = 2$.

$$f(2) = 3(2)^4 - 5(2) + \sqrt[3]{(2)^2 + 4} = 3(16) - 10 + 2 = 40$$

Calculate the limit as x approaches 2 using the limit laws.

$$\begin{aligned}
\lim_{x \to 2} f(x) &= \lim_{x \to 2} \left(3x^4 - 5x + \sqrt[3]{x^2 + 4}\right) \\
&= \lim_{x \to 2} 3x^4 - \lim_{x \to 2} 5x + \lim_{x \to 2} \sqrt[3]{x^2 + 4} \\
&= 3 \lim_{x \to 2} x^4 - 5 \lim_{x \to 2} x + \sqrt[3]{\lim_{x \to 2} (x^2 + 4)} \\
&= 3 \left(\lim_{x \to 2} x\right) \left(\lim_{x \to 2} x\right) \left(\lim_{x \to 2} x\right) \left(\lim_{x \to 2} x\right) - 5 \left(\lim_{x \to 2} x\right) + \sqrt[3]{\lim_{x \to 2} x^2 + \lim_{x \to 2} 4} \\
&= 3 \left(\lim_{x \to 2} x\right)^4 - 5(2) + \sqrt[3]{\left(\lim_{x \to 2} x\right) \left(\lim_{x \to 2} x\right) + 4} \\
&= 3(2)^4 - 10 + \sqrt[3]{(2)(2) + 4} \\
&= 3(16) - 10 + \sqrt[3]{8} \\
&= 48 - 10 + 2 \\
&= 40
\end{aligned}$$

The condition in the definition is satisfied, so $f(x) = 3x^4 - 5x + \sqrt[3]{x^2 + 4}$ is a continuous function at $a = 2$.